Conformal geometric algebra and sphere fitting applied to colour image segmentation

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Abstract. A common task in computer vision is to segment regions based in its colour. Over the years several algorithms have been proposed, ranging from thresholding to more sophisticated algorithms such as belief propagation. In this paper, conformal geometric algebra is used to fit spheres to a set of points representing colours of some colour space, showing how these spheres can be used to identify pixels in images that belong to the covering spheres, i.e segment regions based in its colour.

1 Introduction

In general, the conformal model can be seen as an extension of the Euclidean space R^n, where the conformal model is generated by $\{e_o, e_\infty, e_1, .., e_n\}$, where $e_o \cdot e_i = e_\infty \cdot e_i = 0$, $e_o^2 = e_\infty^2 = 0$ and $e_o \cdot e_\infty = -1$. In the conformal domain a Euclidean point x is represented as:

$$X = x + \frac{1}{2}x^2 e_\infty + e_o \tag{1}$$

and spheres are represented by:

$$S = C - \frac{1}{2}\gamma^2 e_\infty \tag{2}$$

where C is a conformal point that represents the centre of S and γ is the radius.

In the conformal model distance from a point to a sphere is given by:

$$X \cdot S = \gamma^2 - (x - c)^2 \tag{3}$$

Eq.(3) is equivalent to the concept of "power" between a point and sphere, which is used in the context of covering spheres to know how much a point belongs to a sphere.

From this point of view, the segmentation of images based on its colour can

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be expressed as fitting a sphere S to a set X of colours that are similar, for instance a certain tone of red.

Stating the problem in a geometric way makes possible to solve it by just computing the "power" between the S and some point Y, fig.(1) illustrates three different situations of how spheres can be used to segment colour images.

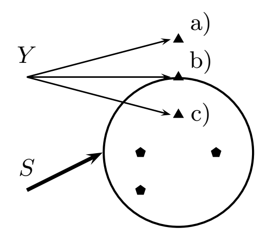


Fig. 1. Three possible situations used to segment regions based on its colour

where:

a) if $Y \cdot S < 0$ then Y is outside S.
b) if $Y \cdot S = 0$ then Y lies on S.
c) if $Y \cdot S > 0$ then Y is inside S.

one clear restriction is that colours contained by S must be similar, because using colours which are very different would result in a sphere that would not be useful. In such situation it is preferable to use several spheres containing different colours.

2 Fitting spheres to colour points

In recent years, treating the colour pixels as points in R^3 regardless of their colour space has been used for gradient detection [1], [2], segmentation [3], and colour representation [5]. Representing colour pixels in such a way makes the problem of segmentation suitable to be stated as a geometric problem, where Geometric Algebra (GA) becomes a powerful tool.

Fitting a sphere S to some $X \subset R^n$ consists in finding S such that it optimally covers all points in X. From this statement, spheres become an interesting option to represent colours which are "similar".

This concept can also be used to fit a sphere to a set of colour pixels $X = \{X_1, ..., X_m\} \in R^3$ using a least square approach:

$$S_* = \underset{S}{\operatorname{argmin}} \sum_{i=1}^{m} (X_i \cdot S)^2 \tag{4}$$

where $X_i \cdot S$ can be represented in a matrix form as follows

$$W = \begin{bmatrix} X_1^1 & X_1^2 & X_1^3 & -1 & \frac{1}{2} X_1 \cdot X_1 \\ X_2^1 & X_2^2 & X_2^3 & -1 & \frac{1}{2} X_2 \cdot X_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ X_n^1 & X_n^2 & X_n^3 & -1 & \frac{1}{2} X_n \cdot X_n \end{bmatrix} \tag{5}$$

where X_i^j represents the j-th component of the i-th point in $X \in R^3$, eq.(5) can be solved by computing the eigen-values, and eigen-vectors [6] of $B = W'W$.

Once S has been computed it can be used to determine whether some point Y is found inside S, this is done in the following way:

- if $Y \cdot S \geq 0$ then Y is inside S.
- if $Y \cdot S < 0$ then Y is outside S.

an advantage of computing S is that it can be used on different images to segment colours. It should be noted that sometimes a point Y that is similar to those in X could be outside S, in which case it is preferable to use a threshold t.

3 Experimental results

In this section we show to different experiments of how the proposed solution works. First we use the well known image of peppers to segment regions of similar colour, then using real image of human retina, the optic disk is segmented. The second experiment consists in segmenting objects of different colours.

Fig.(2(a)), shows the image before segmentation, in order to create a sphere S a set of colours X must be created, this is done manually by selecting some point from the area of interest, in this experiment only four samples where taken from the red pepper (right side in fig.2(a)), the result of the segmentation process using the different thresholds are shown in fig.(3).

(a) (b) (c) (d)

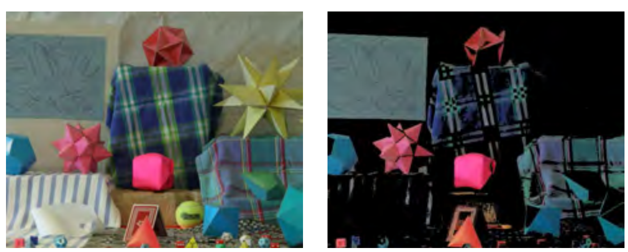
Fig. 2. Result of using different thresholds -0.1, -0.05 and 0.0 for images (b) , (c) and (d) respectively

the proposed solution was also used to segment the optic disk from human retina images, the images were also sampled over the optic disk, fig.(3), only five samples were taken to create a sphere before segmenting the optic disk.

(a) (b) (c)

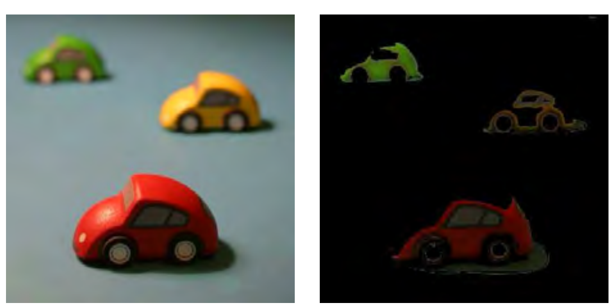
Fig. 3. (a) original image, (b) segmentation using a threshold of -0.1 and (c) segmentation using a threshold of -0.2

The second experiment consisted in segmenting objects of different colours, unlike the previous experiment where only one sphere was used,in this epxeriment several spheres corresponding to various colours are used, fig.(3), five samples per colour are taken before segmenting a region of the selected colours, then separate spheres are created for each selected colour.



(a) Original image

(b) Two spheres used to segment blue and pink



(c) Original image

(d) Two spheres used to segment red and green

Fig. 4. Result of different spheres for colour segmentation

This last experiment had some problems segmenting different regions simultaneously, because colours where relatively close. This situations favoured negative thresholds, which lead to situations such as in fig.(4(d)), where yellow colour was segmented when only red and green were intended to be segmented.

In these two experiments we have choosen to take only four or five samples of a given colour, it is not a restriction of the proposed method. However, it should be noted that is peferable to select colours that are very similar.

4 Future work

The method presented in this paper has the limitation of using spheres that contain colours that must be similar, which sometimes results in difficulties to establish a reliable threshold.Therefore one task that will be addressed in the future it to determine a maximum and minimum threshold, which should take into account the properties of the spheres that represent some colour. It would also be interesting to adapt our proposed solution to other segmenting tasks such as stereo matching in multiple views and motion tracking.

5 Conclusion

In this paper it has been shown hos the colour segmentation problem can be stated in geometric way, and the useful tool GA can be in solving problems of computer vision. It must also be noted that using the proposed solution to segment real images its a good option since it allows to take samples from different images and use the computed sphere on images that were not used to take samples.

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